

Cosmological Parameters from a re-analysis of the WMAP-7 low resolution maps

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ABSTRACT

Cosmological parameters from WMAP 7 year data are re-analyzed by substituting a pixel-based likelihood estimator to the one delivered publicly by the WMAP team. Our pixel based estimator handles exactly intensity and polarization in a joint manner, allowing to use low-resolution maps and noise covariance matrices in T, Q, U at the same resolution, which in this work is $N_{\text{side}} = 16$. We describe the features and the performances of the code implementing our pixel-based likelihood estimator. We perform a battery of tests on the application of our pixel based likelihood routine to WMAP publicly available low resolution foreground cleaned products, in combination with the WMAP high- ℓ likelihood, reporting the differences on cosmological parameters evaluated by the full WMAP likelihood public package. The credible central value for the cosmological parameters change below the 1σ level with respect to the evaluation by the full WMAP 7 year likelihood code, with the largest difference in a shift to smaller values of the scalar spectral index n_s .

Key words: Cosmology: cosmic microwave background - Physical data and processes: cosmological parameters.

1 INTRODUCTION

The anisotropy pattern of the cosmic microwave background (CMB) is a treasure for understanding the constituents of our Universe and how it evolved from the Big Bang. Under the assumption of isotropy and Gaussianity of CMB fluctuations, the power spectra of intensity and polarization anisotropies include all the compressed information on our Universe through the determination of the cosmological parameters. There has been a tremendous improvement in the estimate of cosmological parameters driven by the increasingly better quality of CMB data, mainly due to the full sky observations in temperature and polarization by the Wilkinson Microwave Anisotropy Probe (WMAP), (see Larson et al. (2010); Komatsu et al. (2010) and references therein) and to the small angular scales measurements by QUAD in polarization (Brown et al. 2009), by SPT (Lueker et al. 2010; Keisler et al. 2011; Reichardt et al. 2012) and ACT (Das et al. 2011; Dunkley et al. 2011) in temperature. PLANCK will lead to a drastic improvement of CMB full sky maps in temperature and polarization,

leading to an eagerly expected improvement in cosmological parameters with uncertainties at the percent level (Planck Collaboration 2005).

A joint likelihood analysis in temperature and polarization is one of the accepted methods in securing the scientific expectations of observational achievements in terms of cosmological parameters. Although the likelihood could be written exactly in the map domain under the Gaussian hypothesis, its computation is almost prohibitive already at the resolution of 2 degrees, whereas cosmological information is encoded in the temperature and polarization power spectra up to the angular scales of the order of few arcminutes, where the Silk damping suppress the CMB primary anisotropy spectrum. It is now commonly accepted to use an hybrid approach which combines a pixel approach at low resolution with an approximated likelihood based on power spectrum estimates at high multipoles (see Bond, Jaffe and Knox (2000); Verde et al. (2003); Hamimeche and Lewis (2008) for some of these approximations).

Since the three year release of the full polarization information, the WMAP team adopted such a hybrid scheme approach, which has been suggested independently by Efstathiou (2004); Slosar, Seljak and Makarov (2004); O’Dwyer et al. (2004). At a first appearance of the three

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year data, the WMAP team adopted a pixel approach on HEALPIX (Gorski et al. 2005) resolution $N_{\text{side}} = 8$ ¹ temperature and polarization maps, and considered the high- ℓ approximated likelihood to start at $\ell = 13$ in temperature and $\ell = 24$ in polarization and temperature-polarization cross-correlation for the determination of cosmological parameters in Spergel et al. (2007). The WMAP team treats separately temperature and polarization as explained in Page et al. (2007) and Hinshaw et al. (2007), by using the approximation that the noise in temperature is negligible. As a consequence, the WMAP likelihood code includes either (Q, U) and the temperature-polarization cross-correlation in the same sub-matrix. It was then shown by Eriksen et al. (2007) that by increasing the resolution of the temperature map to HEALPIX $N_{\text{side}} = 16$ and therefore the multipole of transition to high- ℓ approximated likelihood in temperature from $\ell = 12$ to $\ell = 30$, the mean value for the scalar spectral index n_s shifted to higher values by a 0.4σ . The asymmetric handling of the low-resolution temperature map at $N_{\text{side}} = 16$ and polarization at $N_{\text{side}} = 8$, became the final treatment of the three year data release. This low- ℓ likelihood aspect in the WMAP hybrid approach has not changed since the final release of the WMAP 3 year data to the current WMAP 7 year one.

In this paper we wish to perform an alternative determination of the cosmological parameters from WMAP 7 public data, substituting the WMAP low- ℓ likelihood approach with a pixel based likelihood code which treats T, Q, U at the same HEALPIX resolution $N_{\text{side}} = 16$ connected to the standard WMAP high- ℓ package. In this analysis we therefore increase the resolution of polarization products digested by the pixel base likelihood from $N_{\text{side}} = 8$ to $N_{\text{side}} = 16$, in analogy with what done by Eriksen et al. (2007) for temperature only. The WMAP 7 year foreground cleaned (Q, U) maps, covariance matrices and masks at the resolution $N_{\text{side}} = 16$ are also publicly available at <http://lambda.gsfc.nasa.gov>; therefore, all data used in this paper are made available by the WMAP team.

The paper is organized as follows. In Section II we briefly describe the WMAP hybrid approach to the likelihood, with particular care to the low multipole part. In Section III we describe our pixel approach, implemented in the BoPix code. We then present in Section IV the cosmological parameters obtained by using our alternative pixel approach in place of the WMAP one for a Λ CDM scenario. In Section V we extend our investigations to other cosmological models. In Section VI we draw our conclusions.

2 A BRIEF DESCRIPTION OF THE WMAP HYBRID LIKELIHOOD ANALYSIS

In the map domain, the likelihood as function of the cosmological parameters $\{\theta\}$

$$\mathcal{L}(\mathbf{d}|\theta) = \frac{1}{|2\pi\mathbf{C}|^{1/2}} \exp \left[-\frac{1}{2} \mathbf{d}^t \mathbf{C}^{-1} \mathbf{d} \right] \quad (1)$$

where the data, $\mathbf{d} = \mathbf{s} + \mathbf{n}$, is a CMB fully polarized map, considered as a vector combining T, Q and U maps, the sum

of signal \mathbf{s} and noise \mathbf{n} ; the quantity $\mathbf{C} = \mathbf{S} + \mathbf{N}$ is the total covariance matrix, the sum of the CMB signal covariance matrix $\mathbf{S}(\theta)$, and the noise matrix \mathbf{N} . The signal covariance is determined by the power spectra C_ℓ^{XY} , where X, Y are any of T, E, B as given in Tegmark and de Oliveira-Costa (2001).

The WMAP low- ℓ likelihood is described in the Appendix of Page et al. (2007) and we report here the essentials. The WMAP approach is based on the assumption to ignore the noise in temperature, which leads to a simplification of the likelihood, useful from the numerical computation perspective. By assuming that the noise in temperature is negligible at low multipoles, the WMAP approach consists in rewriting Eq. (1) as:

$$\mathcal{L}(\mathbf{d}|\theta) \simeq \frac{\exp \left(-\frac{1}{2} \mathbf{s}_T^t S_T^{-1} \mathbf{s}_T \right)}{\sqrt{2\pi} |S_T|^{1/2}} \times \frac{\exp \left[-\frac{1}{2} \tilde{\mathbf{d}}_P^t (\tilde{S}_P + \tilde{N}_P)^{-1} \tilde{\mathbf{d}}_P \right]}{|\tilde{S}_P + \tilde{N}_P|^{1/2}} \quad (2)$$

where S_T is the temperature signal sub-matrix, the new polarization data vector is $\tilde{\mathbf{d}}_P = \tilde{\mathbf{s}}_P + \tilde{\mathbf{n}}_P$, with $\tilde{\mathbf{s}}_P = (\tilde{Q}, \tilde{U})$ given by

$$\tilde{Q} \equiv Q - \frac{1}{2} \sum_{l=2}^{\ell_P} \frac{C_l^{TE}}{C_l^{TT}} \sum_{m=-l}^l a_{\ell m}^{TT} (+_2 Y_{\ell m} - {}_{-2} Y_{\ell m}^*), \quad (3)$$

$$\tilde{U} \equiv U - \frac{i}{2} \sum_{l=2}^{\ell_P} \frac{C_l^{TE}}{C_l^{TT}} \sum_{m=-l}^l a_{\ell m}^{TT} (+_2 Y_{\ell m} - {}_{-2} Y_{\ell m}^*), \quad (4)$$

with \tilde{S}_P (\tilde{N}_P) is the signal (noise) covariance matrix for the new polarization vector (Page et al. 2007). The noise covariance matrix for (\tilde{Q}, \tilde{U}) equals the original one for (Q, U) when the noise in temperature is zero (Page et al. 2007). As temperature $a_{\ell m}^{TT}$, the full-sky internal linear combination (ILC) map is used (Hinshaw et al. 2007).

According to Page et al. (2007), Eq. (1) and Eq. (2) are mathematically equivalent when the temperature noise is ignored. With this assumption, the new form, Eq. (2), allows the WMAP approach to factorize the likelihood of temperature and polarization, with the information in their cross-correlation, C_ℓ^{TE} , retained in the polarization sub-matrix. As already mentioned in the introduction, temperature is considered at the HEALPIX resolution $N_{\text{side}} = 16$, whereas polarization at $N_{\text{side}} = 8$. The range of multipoles considered in the polarization sub-matrix is up to the Nyquist limit at $N_{\text{side}} = 8$, i.e. $\ell_P = 23$. Two computation options are available for the temperature likelihood, Gibbs sampling (Jewell, Levin and Anderson 2004; Wandelt, Larson and Lakshminarayanan 2004; Eriksen et al. 2004) with a range of multipole considered up to $\ell_T = 32$ and pixel base evaluation, with $\ell_T = 30$.

The high- ℓ likelihood, described in Larson et al. (2010) and in Verde et al. (2003), has been updated to beam/point sources uncertainties through the various subsequent WMAP releases (Hinshaw et al. 2007; Nolte et al. 2009). The high- ℓ TT likelihood takes into account multipoles from $\ell = 31$ ($\ell = 33$) when connected with the pixel (Gibbs) likelihood evaluation of the low resolution temperature data up to $\ell = 1200$; the high- ℓ TE (and TB when used) likelihood takes into account multipoles from $\ell = 24$ (Page et al. 2007)

¹ The number of pixels in a map is given by $N_{\text{pix}} = 12N_{\text{side}}^2$, i.e. 768 for $N_{\text{side}} = 8$ and 3072 for $N_{\text{side}} = 16$.

to $\ell = 800$. High- ℓ EE and BB data have not used so far in the various releases of the WMAP likelihood code.

3 BOPIX

BoPix computes the likelihood function in Eq. (1) for the parameter space $\{\theta\}$ which the $C_\ell^{XY}(\{\theta\})$ depend on, *without any approximation and with the same resolution in temperature and polarization*. BoPix is a multithreaded OpenMP Fortran90 library which can be connected to a sampler - to CosmoMC Lewis and Bridle (2002) in this work.

The computation of the likelihood given in Eq. (1) requires an environment initialization, in which BoPix calculates the geometrical functions dependent on the cosine of the angle between two pixels and reads the noise covariance matrix (C-binary format).

BoPix then starts to compute the signal covariance matrix \mathbf{S} for a given $C_\ell^{XY}(\{\theta\})$ with an OpenMP routine with a high intrinsic level of parallel architecture, to which the noise covariance matrix \mathbf{N} is summed. The full covariance matrix is then Cholesky decomposed. The computation of the determinant is obtained from the properties of the Cholesky decomposed matrix \mathbf{L} : $\det \mathbf{C} = (\det \mathbf{L})^2$. The term $\mathbf{C}^{-1}\mathbf{d}$ is computed as the solution for the variable \mathbf{x} (vector with dimension $3N_{\text{pix}}$) of the equation $\mathbf{C}\mathbf{x} = \mathbf{d}$.

The matrix manipulations are implemented on LAPACK and BLAS mathematical libraries (as nag, essl, acml and mkl). There is an effort to improve the BoPix capabilities and performances (in terms of run time and memory) to make the direct likelihood evaluation at low resolution for cosmological parameters extraction as fast as possible, in particular by reducing the time spent for the Cholesky decomposition, and optimizing the combined scalability in memory and CPU time of this code; indeed, the resources required by BoPix are larger than those for the WMAP low- ℓ likelihood code since the polarization sector is treated at higher resolution. At present, BoPix can handle maps and full noise covariances up to HEALPIX $N_{\text{side}} = 32$ resolution. On IBM Power6 (4.2GHz) architecture, available at CINECA (<http://www.cineca.it>), with 64 threads on 64 logical CPUs (32 cores) BoPix can calculate the likelihood in about 0.3 seconds at $N_{\text{side}} = 16$, and in about 15 seconds at $N_{\text{side}} = 32$. At $N_{\text{side}} = 16$ on the same IBM Power6, a good trade off between computation time and memory required is obtained for 2 sec with 8 cores. More details about performances and comparison among different platforms will be provided in De Rosa (2012).

4 DATA SET FOR BOPIX

We use the temperature ILC map smoothed at 9.8125 degrees and reconstructed at HealPix (Gorski et al. 2005) resolution $N_{\text{side}} = 16$, the foreground cleaned (unsmoothed) low resolution maps and the noise covariance matrix in (Q, U) publicly available at the LAMBDA website <http://lambda.gsfc.nasa.gov/> for the frequency channels Ka (23 GHz), Q (41GHz) and V (61 GHz) as considered by Larson et al. (2010) for the low ℓ analysis. These frequency channels have been co-added by inverse noise covariance

weighting accordingly to the WMAP team (Jarosik et al. 2007)

$$\mathbf{d}_{\text{pol}} = \mathbf{c}_{\text{pol}}(\mathbf{c}_{Ka}^{-1}\mathbf{d}_{Ka} + \mathbf{c}_Q^{-1}\mathbf{d}_Q + \mathbf{c}_V^{-1}\mathbf{d}_V), \quad (5)$$

where \mathbf{d}_i , \mathbf{c}_i are the polarization maps and covariances, respectively (for $i=Ka, Q$ and V). The total noise covariance matrix is therefore:

$$\mathbf{c}_{\text{pol}}^{-1} = \mathbf{c}_{Ka}^{-1} + \mathbf{c}_Q^{-1} + \mathbf{c}_V^{-1}. \quad (6)$$

This polarization data set has been extended to temperature considering the ILC map with an extra noise term, as suggested in Dunkley et al. (2009). We have therefore added to the temperature map a random noise realization with variance of $\sigma_{TT}^2 = 1\mu K^2$ and consistently, the noise covariance matrix for TT is taken to be diagonal with variance equal to $1\mu K^2$. The total noise covariance \mathbf{N} for WMAP 7 yr data is therefore:

$$\mathbf{N} = \begin{pmatrix} \sigma_{TT}^2 \mathbf{I} & 0 \\ 0 & \mathbf{c}_{\text{pol}} \end{pmatrix}$$

Let us note that this prescription of the noise in the temperature ILC map added to mitigate the uncertainties due to foreground cleaning violates the assumption that the noise in temperature is vanishing, used to obtain Eqs. (2,3,3) from Eq. (1).

Two masks are considered: one for T and one for Q and U. Monopole and dipole have been subtracted from the observed ILC map through the HealPix routine REMOVE-DIPOLE (Gorski et al. 2005). The same data set has been used for the WMAP 7 yr power spectrum re-analysis by the QML estimator BolPol in Gruppuso et al. (2009) (and further used in Paci et al. (2010); Gruppuso et al. (2011)).

5 COSMOLOGICAL PARAMETERS EXTRACTION

We use CosmoMC (Lewis and Bridle 2002) in order to compute the Bayesian probability distribution of model parameters. The pivot scale of the primordial scalar and tensor power spectra was set to $k_* = 0.017 \text{ Mpc}^{-1}$, as recommended by Cortes, Liddle and Mukherjee (2009). We vary the physical baryon density $\Omega_b h^2$, the physical cold dark matter density $\Omega_c h^2$, the ratio of the sound horizon to the angular diameter distance at decoupling θ , the reionisation optical depth τ , the amplitude and spectral index of curvature perturbations n_s and $\log_{10}[10^{10} A_s]$. We assume a flat universe, and so the cosmological constant for each model is given by the combination $\Omega_\Lambda = 1 - \Omega_b - \Omega_c$. We set the CMB temperature $T_{\text{CMB}} = 2.725 \text{ K}$ (Mather et al. 1999) and the primordial helium fraction to $y_{\text{He}} = 0.24$. We assume three neutrinos with a negligible mass. In order to fit WMAP data, we use the lensed CMB and we follow the method implemented in CosmoMC consisting in varying a nuisance parameter A_{SZ} which accounts for the unknown amplitude of the thermal SZ contribution to the small-scale CMB data points assuming the model of . We use a CAMB (Lewis, Challinor and Lasenby 2000) accuracy setting of 1. We sample the posterior using the Metropolis-Hastings algorithm (Hastings 1970) at a temperature $T = 1$, generating four parallel chains and imposing a conservative Gelman-

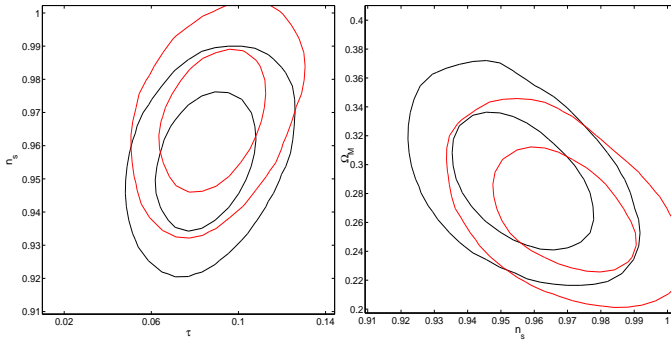


Figure 1. Marginalized 68% and 95%-credible contours for (τ, n_s) (left panel) and (n_s, Ω_M) (right panel) as estimated by the WMAP 7 year full likelihood (red lines) and by the BoPix plus WMAP 7 year high ℓ likelihood (black lines).

Rubin convergence criterion (Gelman and Rubin 1992) of $R - 1 < 0.005$.

With the settings specified above we extract cosmological parameters with the latest WMAP likelihood code (version v4p1) available at <http://lambda.gsfc.nasa.gov/> as benchmarks. We prefer to not quote the estimates for the cosmological parameters performed by the WMAP team since the conventions and the CAMB version might differ from those used in Larson et al. (2010); Komatsu et al. (2010).

We then extract cosmological parameters by substituting the WMAP low- ℓ likelihood approach with BoPix. In doing this we implicitly use the WMAP inputs in polarization at $N_{\text{side}} = 16$ as described in Section III and not those contained in the WMAP likelihood routine publicly available. Since temperature and polarization are treated at the same resolution by BoPix, we include the WMAP high ℓ likelihood starting at $\ell = 31$ both in temperature and temperature-polarization cross-correlation when using BoPix, unless otherwise stated. Unless otherwise stated, in BoPix we vary the C_ℓ up to $\ell = 30$ and we use the publicly available file *test_cls_v4.dat* as a fiducial power spectrum to complete the full covariance at low resolution from $\ell = 31$ to $\ell = 64$, as done for temperature only by the WMAP pixel likelihood.

We find small differences in the estimate of the cosmological parameters by substituting BoPix to the WMAP low- ℓ likelihood, as reported in Table I.² The main difference between the estimate of the cosmological parameters derived by our alternative low- ℓ likelihood code and the one obtained with the WMAP approach is in the spectral index n_s : we obtain a value for n_s which is 0.86σ lower than the WMAP one. This change would lead to quantitative differences in the evidence against the Harrison-Zeldovich of the WMAP 7 yr data. However, also the other directly sampled

cosmological parameters differ from the WMAP estimate in about 0.5σ , pointing towards values higher for the physical CDM abundance $\Omega_c h^2$ and the amplitude of scalar perturbations A_S and smaller for the baryon physical content $\Omega_b h^2$ and optical depth τ . As a derived parameters, we have a higher value for the matter content Ω_M and σ_8 , smaller for the present Hubble rate H_0 . We show more details about these different estimates in the two-dimensional plots of Fig. 5. These differences seems robust to the change in the multipole transition to the high likelihood approximation and to the change of the fiducial model to complete the covariance at low resolution. Special mention should be made for the case in which we do not consider $\ell_T = \ell_P$, but we adopt the same $\ell_T = 30$ and $\ell_P = 23$ adopted by the WMAP team, but with BoPix for low resolution: the differences with respect to the estimates by the full WMAP yr likelihood are slightly smaller than in the case of $\ell_T = \ell_P = 30$, as can be seen in Table I. This means that differences we find are not fully due to the different threshold multipoles for polarization adopted in the two low- ℓ likelihood approaches. No appreciable differences are noticed by constructing the signal covariance matrix up to $3N_{\text{side}}$ instead up to $4N_{\text{side}}$. This can be understood since this different prescription in constructing the signal covariance matrix is damped by the Gaussian smoothing in intensity and is much below the noise in polarization.

We have performed a further test excluding A_{SZ} , just for code comparison. We find a smaller discrepancy between the estimates for the cosmological parameters and the best-fits from the two likelihood approaches when the nuisance parameter A_{SZ} is omitted (i.e. fixed to zero). This additional foreground parameter A_{SZ} is not well constrained by WMAP, but it contributes to the shape of the final likelihood and to the marginalized values of the parameters (shifting slightly the value of n_s , for instance).

Most of these small differences reported in the estimate of the cosmological parameters interfere destructively because of the cosmic confusion (Efsthathiou and Bond 1998) and the best-fits C_ℓ from the two likelihood analysis agree very well. We present the CMB bestfit C_ℓ^{TT} obtained by BoPix in combination with the WMAP 7 high- ℓ likelihood in comparison with those obtained by the full WMAP 7 likelihood in Fig. 5. The difference in the best-fit C_ℓ is consistent with the different central values for the cosmological parameters displayed in Table 5. We have checked that the best-fit C_ℓ obtained in this work by the full WMAP 7 likelihood has $\Delta(-2 \log \mathcal{L}_{\text{WMAP}}) = -7.42$ with respect to the reference WMAP 7 *test_cls_v4.dat*; the best-fit C_ℓ obtained in this work by BoPix in combination with the high- ℓ WMAP 7 likelihood provides a better fit, with $\Delta(-2 \log \mathcal{L}_{\text{WMAP}}) = -7.75$ with respect to the reference WMAP 7 *test_cls_v4.dat*.

Since the pixel approach and the Gibbs approach contained in the WMAP likelihood package give quite consistent results, the difference we find by using BoPix is more likely due to the asymmetric treatment of T and (Q, U) adopted in WMAP low- ℓ likelihood. We have therefore tested BoPix against the WMAP likelihood within the same range of multipole, i.e. up to $\ell = 30$: BoPix has been run on the low-resolution WMAP 7 yr $N_{\text{side}} = 16$ products varying C_ℓ^{TT} , C_ℓ^{EE} , C_ℓ^{TE} up to $\ell = 30$ and compared to the likelihood obtained by the WMAP 7 yr pixel based routine plus the high- ℓ likelihood value for TE from $\ell = 24$ to $\ell = 30$. In this way

² Note that the small differences of our results with the full WMAP 7 year likelihood with respect to the results reported by Larson et al. (2010) or Komatsu et al. (2010) might be ascribed to the different version of RECFAST used, different tools for extracting cosmological parameters or different conventions, such as the pivot scale k_* .

Parameter	WMAP 7 likelihood (Pixel)	WMAP 7 likelihood (Gibbs)	$\ell_T = \ell_P = 30$	$\ell_T = \ell_P = 24$	$\ell_T = \ell_P = 36$	$\ell_T = 30$ $\ell_P = 23$	Different Fiducial
$100 \Omega_b h^2$	2.250 ± 0.056	$2.252^{+0.057}_{-0.056}$	2.213 ± 0.055	2.215 ± 0.055	$2.224^{+0.057}_{-0.058}$	2.224 ± 0.055	2.212 ± 0.058
$\Omega_c h^2$	$0.1114^{+0.0054}_{-0.0053}$	0.1114 ± 0.0055	$0.1145^{+0.0055}_{-0.0056}$	0.1142 ± 0.0055	$0.1152^{+0.0055}_{-0.0056}$	$0.1123^{+0.0058}_{-0.0057}$	0.1144 ± 0.0056
τ	0.089 ± 0.015	$0.089^{+0.014}_{-0.015}$	$0.085^{+0.015}_{-0.014}$	$0.085^{+0.015}_{-0.014}$	$0.085^{+0.015}_{-0.014}$	0.087 ± 0.015	0.085 ± 0.015
n_s	$0.968^{+0.014}_{-0.013}$	$0.969^{+0.013}_{-0.014}$	0.956 ± 0.014	$0.957^{+0.014}_{-0.013}$	$0.954^{+0.013}_{-0.014}$	0.959 ± 0.014	$0.956^{+0.014}_{-0.013}$
$\log[10^{10} A_s]$	$3.116^{+0.033}_{-0.032}$	3.116 ± 0.033	3.130 ± 0.033	3.128 ± 0.032	$3.133^{+0.032}_{-0.033}$	3.122 ± 0.033	3.128 ± 0.033
Ω_M	$0.270^{+0.027}_{-0.028}$	0.269 ± 0.028	$0.289^{+0.031}_{-0.030}$	$0.288^{+0.030}_{-0.031}$	0.294 ± 0.031	$0.277^{+0.031}_{-0.030}$	0.289 ± 0.030
H_0	$70.7^{+2.4}_{-2.5}$	$70.7^{+2.5}_{-2.4}$	$68.9^{+2.3}_{-2.4}$	69.1 ± 2.4	68.5 ± 2.4	69.9 ± 2.5	$68.9^{+2.3}_{-2.4}$
σ_8	0.811 ± 0.029	0.811 ± 0.029	0.820 ± 0.029	$0.819^{+0.028}_{-0.029}$	$0.822^{+0.030}_{-0.028}$	$0.810^{+0.031}_{-0.030}$	$0.820^{+0.029}_{-0.030}$

Table 1. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters estimated by the WMAP 7 year full likelihood (second and third column) and by the BoPix plus WMAP 7 year high ℓ likelihood for different transition multipoles $\ell_T = \ell_P$ (fourth, fifth and sixth column), for $\ell_T \neq \ell_P$ and different fiducial theoretical power spectrum to complete the signal covariance matrix in BoPix (last column). Below the thick line analogous mean values and bounds are presented for derived parameters.

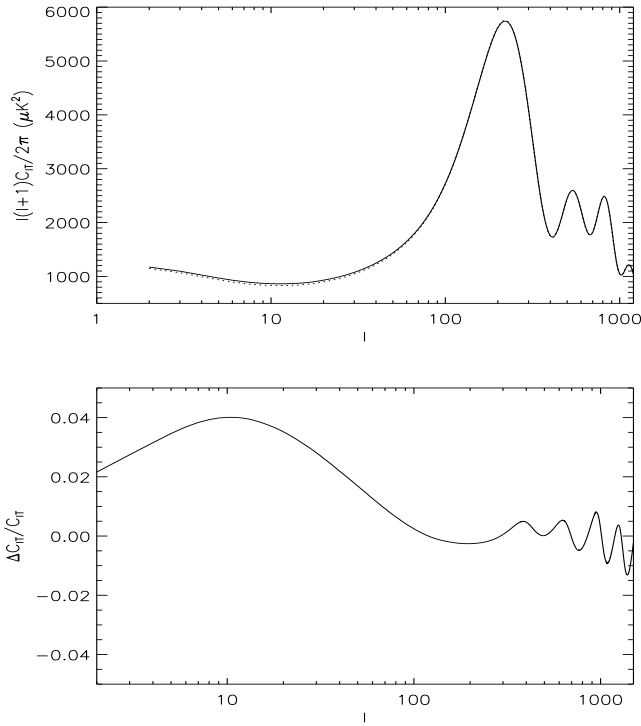


Figure 2. Comparison of the best-fit C_ℓ obtained by BoPix in combination with the WMAP 7 high- ℓ likelihood (solid) vs. the WMAP 7 full likelihood (dashed) is shown in the top panel. To make the difference more visible, the relative difference between the C_ℓ bestfits in temperature is shown in the bottom panel. Note that the differences are well within the cosmic variance.

we subtract the *same* high- ℓ likelihood information from hybrid runs presented in Table I. By assuming $\Omega_b h^2 = 0.02246$, $\Omega_c h^2 = 0.1117$ and sound horizon $\theta = 1.03965$, we obtain results quite consistent with the hybrid ones: a slight smaller value in the estimate of τ and n_s and a larger one for A_s .

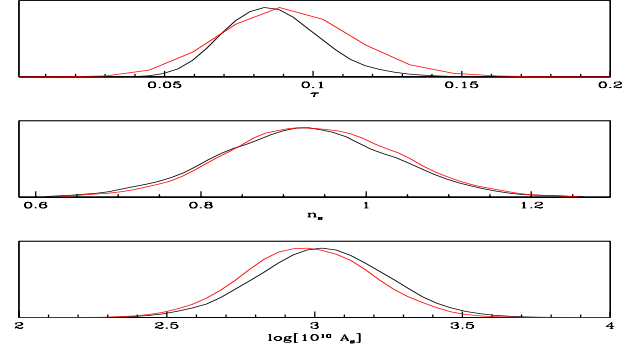


Figure 3. Marginalized one-dimensional probabilities for τ , n_s and $\log[10^{10} A_s]$ as estimated by the WMAP 7 year full likelihood (red lines) and by the BoPix plus WMAP 7 year high ℓ likelihood (black lines). See text for further details.

6 OTHER EXTENDED COSMOLOGICAL MODELS

We now consider few cosmological models beyond the Λ CDM model which can be constrained by WMAP 7 year data only. We consider only the baseline $l_{\text{trans}} = 30$ and all the other settings consistently with the previous section, unless otherwise stated.

Gravitational Waves.

We consider all inflationary models which can be described by the primordial perturbation parameters consisting of the scalar amplitude and spectral index (A_s, n_s), and the tensor-to-scalar ratio r . In canonical single-field inflation, in the slow-roll limit, the tensor spectrum shape is not independent of the scalar one. We will consider a tensor spectrum with a tilt $n_T = -r/8$, as predicted for canonical single-field inflation at first-order in slow-roll.

Our marginalised 68%-credible interval for the scalar spectral index is given by $n_s = 0.977^{+0.020}_{-0.021}$ (to be compared with the result of the previous section, $n_s = 0.955 \pm 0.014$), half a sigma redder than the result we obtain by the full WMAP 7 year likelihood 0.987 ± 0.020 .

Parameter	WMAP 7 likelihood	BoPix plus WMAP 7 high ℓ likelihood
$100 \Omega_b h^2$	$2.307^{+0.071}_{-0.072}$	2.270 ± 0.073
$\Omega_c h^2$	0.1073 ± 0.0063	$0.1099^{+0.0067}_{-0.0066}$
τ	$0.091^{+0.015}_{-0.014}$	$0.087^{+0.015}_{-0.014}$
n_s	0.987 ± 0.020	$0.977^{+0.020}_{-0.021}$
$\log[10^{10} A_s]$	3.093 ± 0.038	3.102 ± 0.039
r	< 0.34	< 0.36
Ω_M	$0.246^{+0.031}_{-0.032}$	$0.262^{+0.035}_{-0.036}$
H_0	73.2 ± 3.2	$71.6^{+3.2}_{-3.3}$
σ_8	0.797 ± 0.033	0.805 ± 0.033

Table 2. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters including the tensor-to-scalar ratio estimated by the WMAP 7 year full likelihood (left column) and by the BoPix plus WMAP 7 year high ℓ likelihood (right column). For the tensor-to-scalar ratio r the 95%-credible upper bound is quoted. Below the thick line analogous mean values and bounds are presented for derived parameters.

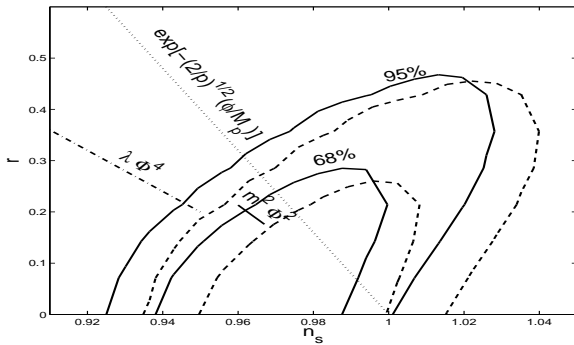


Figure 4. Marginalized 68% 95% contours for (n_s, r) as estimated by the WMAP 7 year full likelihood (dashed lines) and by the BoPix plus WMAP 7 year high ℓ likelihood (solid lines). Theoretical predictions of few popular inflationary models (including reheating uncertainties where appropriate) are displayed.

At 95% confidence level, our result for the tensor-to-scalar ratio is $r < 0.36$, fully consistent with the result we obtain from the full WMAP 7 year likelihood, i.e. $r < 0.34$. Let us note that, differently from the WMAP low- ℓ likelihood code, the value obtained by BoPix include BB polarization in the construction of the covariance at low resolution. The differences in the (n_s, r) are shown in Fig. 2 and are mainly due to a shift of the constraints at smaller values for n_s , as occurs for the standard Λ CDM model discussed in the previous section. Theoretical predictions of few popular inflationary models (including reheating uncertainties where appropriate) are displayed. One of the phenomenological differences from the different constraints would be a minor tension for a massless self-interacting inflaton model with WMAP 7 year data *only* (see Komatsu et al. (2010); Finelli et al. (2010) as examples for an higher tension of the $\lambda\phi^4$ potential with observations when additional cosmological data sets are added to WMAP).

Running of the scalar spectral index.

In this subsection we consider the variation of the scalar

Parameter	WMAP 7 likelihood	BoPix plus WMAP 7 high ℓ likelihood
$100 \Omega_b h^2$	$2.198^{+0.074}_{-0.072}$	2.184 ± 0.081
$\Omega_c h^2$	0.1167 ± 0.0082	$0.1175^{+0.0083}_{-0.0084}$
τ	$0.091^{+0.015}_{-0.016}$	0.087 ± 0.015
n_s	0.961 ± 0.016	$0.953^{+0.015}_{-0.016}$
$\log[10^{10} A_s]$	3.154 ± 0.054	$3.151^{+0.054}_{-0.055}$
n_{run}	$-0.074 < n_{\text{run}} < 0.030$	$-0.065 < n_{\text{run}} < 0.042$
Ω_M	0.303 ± 0.049	$0.310^{+0.050}_{-0.051}$
H_0	$68.2^{+3.7}_{-3.6}$	$67.5^{+3.8}_{-3.7}$
σ_8	$0.823^{+0.033}_{-0.032}$	$0.826^{+0.033}_{-0.032}$

Table 3. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters including the running of the scalar spectral index n_{run} estimated by the WMAP 7 year full likelihood (left column) and by the BoPix plus WMAP 7 year high ℓ likelihood (right column). For the running of the scalar spectral index n_{run} the 95%-credible upper bound is quoted.

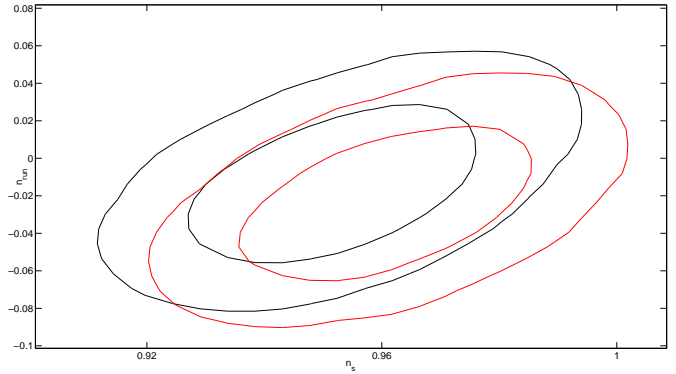


Figure 5. Marginalized 68% and 95%-credible contours for (n_s, n_{run}) as estimated by the WMAP 7 year full likelihood (red lines) and by the BoPix plus WMAP 7 year high ℓ likelihood (black lines).

spectral index with wavelength, i.e. we allow n_{run} to vary in the range $[-0.2, 0.2]$. Our marginalised 95%-credible interval for the scalar spectral index is given by $-0.065 < n_{\text{run}} < 0.042$, which has to be compared with the result we obtain by the full WMAP 7 year likelihood $-0.074 < n_{\text{run}} < 0.030$. The results are fully consistent between each other and with the hypothesis of no wavelength dependence of the scalar spectral index.

Neutrino Mass.

In this subsection we constrain the total mass of neutrinos $\sum m_\nu = 94\Omega_\nu h^2$ eV, allowing to vary the fraction of massive neutrino energy density relative to the total dark matter one $f_\nu = \Omega_\nu/\Omega_{\text{DM}}$. At 95% confidence level, our result for the fraction of massive neutrinos is $f_\nu < 0.113$, whereas we obtain $f_\nu < 0.094$ from the full WMAP 7 year likelihood. The resulting neutrino mass bound at 95% confidence level is $\sum m_\nu < 1.4$ eV, compared to 1.1 eV obtained from the full WMAP 7 year likelihood.

Cosmological Birefringence.

Since one of the main differences between the WMAP low

Parameter	WMAP 7 likelihood	BoPix plus WMAP 7 high ℓ likelihood
$100 \Omega_b h^2$	$2.219^{+0.062}_{-0.060}$	2.174 ± 0.061
$\Omega_c h^2$	$0.1177^{+0.0071}_{-0.0073}$	$0.1226^{+0.0081}_{-0.0080}$
τ	$0.087^{+0.014}_{-0.015}$	0.082 ± 0.014
n_s	0.960 ± 0.016	$0.945^{+0.016}_{-0.017}$
$\log[10^{10} A_s]$	3.120 ± 0.032	3.134 ± 0.033
f_ν	< 0.094	< 0.113
Ω_M	$0.329^{+0.057}_{-0.056}$	$0.374^{+0.075}_{-0.072}$
H_0	$65.7^{+4.3}_{-4.2}$	$62.8^{+4.6}_{-4.7}$
σ_8	$0.712^{+0.073}_{-0.074}$	$0.695^{+0.087}_{-0.083}$
$\sum m_\nu$	$< 1.1 \text{ eV}$	$< 1.4 \text{ eV}$

Table 4. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters including the total mass of the neutrinos estimated by the WMAP 7 year full likelihood (left column) and by the BoPix plus WMAP 7 year high ℓ likelihood (right column). For the total mass of the neutrinos $\sum m_\nu$ the 95%-credible upper bound is quoted.

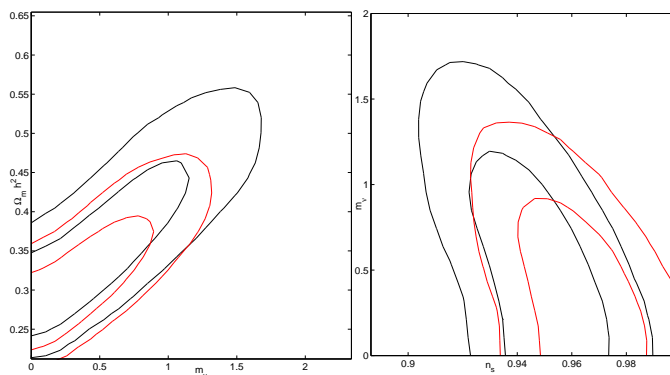


Figure 6. Marginalized 68% and 95%-credible contours for $(\sum_\nu m_\nu, \Omega_M h^2)$ (left panel) and $(n_s, \sum_\nu m_\nu)$ (right panel) as estimated by the WMAP 7 year full likelihood (red lines) and by the BoPix plus WMAP 7 year high ℓ likelihood (black lines).

resolution likelihood code and BoPix is the treatment of the polarization sector, we now wish to analyze an extended cosmological model different from Λ CDM *only* in (Q, U) and the relative cross-correlation with the temperature. Cosmological birefringence refers to a non-vanishing interaction $\propto \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$ between photon and a cosmological evolving pseudo-scalar ϕ , which would generate non-vanishing *TB* and *EB* correlations (Lue, Wang and Kamionkowski 1999) through a rotation α of the polarization plane of CMB photons along their path from the last scattering surface to the observer. The resulting polarization and cross temperature-polarization spectra would encode the particular redshift dependence of the parity violation interaction Finelli and Galaverni (2009). However, a phenomenological shortcut exists, commonly used in the literature and also adopted by the WMAP team, and consists to neglect the redshift dependence of α and simply predict the power spectra as Lue, Wang and Kamionkowski (1999):

$$\begin{aligned} C_\ell^{EE,obs} &= C_\ell^{EE} \cos^2(2\alpha), \\ C_\ell^{BB,obs} &= C_\ell^{EE} \sin^2(2\alpha), \end{aligned}$$

$$C_\ell^{EB,obs} = \frac{1}{2} C_\ell^{EE} \sin(4\alpha), \quad (7)$$

$$C_\ell^{TE,obs} = C_\ell^{TE} \cos(2\alpha),$$

$$C_\ell^{TB,obs} = C_\ell^{TE} \sin(2\alpha).$$

The above formulae are valid when the primordial B-mode polarization is negligible, which is assumed in this paper.

We have therefore sampled α in radians with a flat prior $[-0.5, 0.5]$ plus the other six cosmological parameters of the Λ CDM model by inserting Eqs. (7). Our marginalised 68% (95%)-credible interval for α is $\alpha = -1^\circ.3^{+0^\circ.6+2^\circ.3}_{-0^\circ.7-2^\circ.3}$ in agreement with the full WMAP 7 year likelihood result which we find $\alpha = -1^\circ.0^{+0^\circ.7+2^\circ.4}_{-0^\circ.6-2^\circ.3}$. Either the result using BoPix or the one based on the full WMAP 7 year likelihood are consistent with vanishing cosmological birefringence at 95% CL just by assuming the statistical uncertainty, and the agreement increases by using the systematic uncertainty, which is estimated as $1^\circ.4$ by the WMAP team Komatsu et al. (2010).

Since the weight of the high- ℓ TB likelihood plays a relevant role in these constraints we have also considered the case in which this is not taken into account. Such setting which emphasizes the role of polarization on large angular scales would be relevant to show clearly the potential differences between BoPix and the WMAP pixel likelihood code. On using only low resolution products to constrain cosmological birefringence, by using BoPix on $N_{\text{side}} = 16$ resolution Q, U maps and matrices we obtain $\alpha = -4^\circ.2^{+1^\circ.9+10^\circ.2}_{-3^\circ.1-7^\circ.5}$, still in agreement with the values we find by the WMAP 7 likelihood on $N_{\text{side}} = 8$ resolution Q, U maps and matrices $\alpha = -0^\circ.2^{+3^\circ.6+10^\circ.0}_{-3^\circ.6-9^\circ.9}$. Although with larger uncertainties, our results agree with vanishing cosmological birefringence at 95% CL, without invoking systematic uncertainties. Note also that our result agrees with the analysis on large angular scales by Gruppiso et al. (2012), which has much tighter constraints probably because all the cosmological parameters except α are kept fixed.

The full posterior likelihood for α and its two dimensional contour in combination with the optical depth τ are shown in Fig. (6). Fig. (6) also shows that no degeneracy between τ and α is observed in WMAP 7 yr data. Note that the slight preference at 68% CL for negative values of α when using only BoPix on low resolution products is consistent with the WMAP 7 yr TB and EB power spectra QML estimates at $\ell < 30$ and presented in Gruppiso et al. (2011, 2012).

7 CONCLUSIONS

We have performed an alternative estimate of the cosmological parameters from WMAP 7 year public data, by substituting the WMAP 7 low- ℓ likelihood with a pixel likelihood code which treats (T, Q, U) at the same resolution without any approximation. We have used this code at the HEALPIX resolution $N_{\text{side}} = 16$ on foreground cleaned public data, therefore increasing the resolution of the pixel based polarization products used in our extraction of the cosmological parameters with respect to the WMAP standard one. We have consistently increased the transition multipole from $\ell = 24$ to $\ell = 31$ for the high- ℓ WMAP 7 year temperature-

Parameter	WMAP 7 likelihood	BoPix plus WMAP 7 high ℓ likelihood
$100 \Omega_b h^2$	$2.226^{+0.057}_{-0.055}$	2.217 ± 0.056
$\Omega_c h^2$	$0.1109^{+0.0054}_{-0.0055}$	$0.1145^{+0.0056}_{-0.0055}$
τ	$0.0873^{+0.0147}_{-0.0144}$	$0.0899^{+0.0147}_{-0.0156}$
n_s	0.964 ± 0.014	0.957 ± 0.014
$\log[10^{10} A_s]$	3.191 ± 0.032	3.138 ± 0.033
$\alpha(\text{rad})$	$-0.058 < \alpha < 0.025$	$-0.063 < \alpha < 0.018$
Ω_M	0.270 ± 0.028	$0.290^{+0.030}_{-0.031}$
H_0	70.4 ± 2.4	$68.9^{+2.4}_{-2.5}$
σ_8	0.805 ± 0.029	0.823 ± 0.029

Table 5. Mean parameter values and bounds of the central 68%-credible intervals for the cosmological parameters allowing for an effective treatment of cosmological birefringence estimated by the WMAP 7 year full likelihood (left column) and by the BoPix plus WMAP 7 year high ℓ likelihood (right column). For the angle α defined in Eq. (7) the 95%-credible upper bound is quoted.

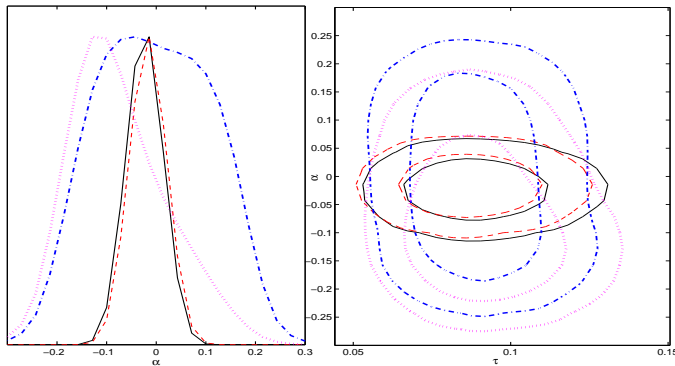


Figure 7. Marginalized posterior probability for α (left panel) and marginalized 68% and 95%-credible contours for (τ, α) (right panel) as estimated by the WMAP 7 year full likelihood (dashed red lines) and by the BoPix plus WMAP 7 year high ℓ likelihood (solid black lines). The additional dot-dashed blue line and short-dashed pink lines are for the constraints on α from large angular scales only obtained by the WMAP 7 year pixel likelihood code and BoPix, respectively.

polarization cross-correlation likelihood and included the marginalization over the nuisance parameter A_{SZ} .

With this setting we have found estimates for the cosmological parameters consistent with those obtained by the full WMAP 7 year likelihood package, although for some parameters the differences are of half σ or more. These differences between the two low- ℓ likelihood treatments we find are larger than the WMAP 7 yr likelihood uncertainties from tests on simulations reported in Larson et al. (2010); however, we need to keep in mind that our differences between two likelihood treatments are reported for *real* data, with WMAP 7 year beam/points source corrections and marginalization on A_{SZ} taken fully into account, differently from the simulation analysis performed in Larson et al. (2010). It is also important to stress that the WMAP assumption to have vanishing noise in temperature is explicitly violated by the ILC WMAP 7 yr temperature map digested by the low- ℓ likelihood code (the likelihood test on simu-

lations reported in Larson et al. (2010) might also miss the addition of the $1\mu\text{K}$ noise per pixel to the low-resolution temperature maps). Another important aspect is that the low resolution polarization data digested by the two likelihood are different since are at different resolution. The difference between the two best-fit C_ℓ for ΛCDM found by the two alternative likelihood treatments show a maximum of 4% around at $\ell \sim 10$ and oscillate with an amplitude below 1% for $\ell > 100$ ³.

On restricting to the ΛCDM model the most important difference is for the scalar spectral index n_s , which decrease to 0.956 from the value 0.968 we obtain with the full WMAP 7 yr likelihood code, i.e. a decrease of 0.86 σ . This different value for n_s would increase the evidence against the Harrison-Zeldovich spectrum from WMAP 7 yr data. This difference for n_s is consistent with the one between the two best-fit C_ℓ and depend only partially from the threshold multipole from which the high- ℓ TE likelihood starts. Other previous alternative likelihood treatments also reported the most important discrepancy for the scalar spectral index (Eriksen et al. 2007; Rudjord et al. 2009). A smaller value for n_s with respect to the estimate by the full WMAP 7 year likelihood code, always within 1 σ , is then seen in all the extension of ΛCDM considered here. No major changes are found for the 95 % credible intervals for the tensor to scalar ratio and for the running of the scalar spectral index. A slight degradation has been found for the 95 % credible interval on the neutrino mass. The case of cosmological birefringence has been taken as a sensitive test for the two alternative likelihoods, whose most relevant difference is the treatment of polarization on large scales. As expected, a slight difference on the posterior of the polarization angle α has been found when only low resolution data are used, whereas the results are fully consistent when the high- ℓ TB data are added to both likelihoods.

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³ We have checked that either the difference between the two best-fit C_ℓ or between the estimates of the cosmological parameters decrease when the nuisance parameter A_{SZ} is set to zero in both alternative likelihood treatments. The net effect of the variation of this foreground parameter, which is unconstrained by the data, is to increase the differences between the estimates of the cosmological parameters from the two likelihood treatments for the ΛCDM model.

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REFERENCES

- Bond J. R., Jaffe A. H., Knox L. E., 2000, *ApJ*, **533**, 19.
- Brown M. L. *et al.* [QUaD Collaboration], 2009, *ApJ*, **705**, 978.
- Cortes M., Liddle A. R., and Mukherjee P., 2007, *Phys. Rev. D*, **75**, 083520.
- Das S. *et al.* [ACT Collaboration], 2011, *ApJ*, 729, 62
- De Rosa A., 2012, in preparation.
- Dunkley J. *et al.* [WMAP Collaboration], *ApJ* SS 180 (2009) 306.
- Dunkley J. *et al.* [ACT Collaboration], 2011, *ApJ*, 739, 52.
- Efstathiou G., 2004, *MNRAS* **349**, 603.
- Efstathiou G., *Mon. Not. Roy. Astron. Soc.* **370**, 343, (2006)
- Efstathiou G., Bond J. R., 1999, *MNRAS* **304** 75
- Eriksen H. K., O’Dwyer I. J., Jewell J. B., Wandelt B. D., Larson D. L., Gorski K. M., Levin S., Banday A. J., Lilje, P.B., *ApJS* **155** 227.
- Eriksen H. K. *et al.*, 2007, *ApJ* **656** 641.
- Finelli F., Galaverni M., *Phys. Rev. D* **79** 063002
- Finelli F., Hamann J., Leach S. M., Lesgourgues, J., 2010, *JCAP* **1004** 011.
- Gelman A., Rubin D. B., 1992, *Statistical Science*, 7, 457.
- Gorski K. M., Hivon E., Banday A.J., Wandelt B.D., Hansen F.K., Reinecke M., Bartelmann M., 2005, *ApJ*, 622, 759.
- Gruppuso A., de Rosa A., Cabella P., Paci F., Finelli F., Natoli P., de Gasperis G., Mandolesi N., 2009, *MNRAS*, **400**, 1.
- Gruppuso A., Finelli F., Natoli P., Paci F., Cabella P., De Rosa A., Mandolesi N., 2011, *MNRAS*, **411**, 1445.
- Gruppuso A., Natoli N., Mandolesi N., De Rosa A., Finelli F., Paci F., 2012, *JCAP* 1202 023
- Hammimeche S., Lewis A., 2008, *Phys. Rev. D*, **77**, 103013.
- Hastings W. K., 1970, *Biometrika*, 57(1), 97.
- Hinshaw G. *et al.* [WMAP Collaboration], 2007, *ApJS* **170** 288
- Jarosik N. *et al.* [WMAP Collaboration], 2007, *ApJS*, **170**, 263.
- Jewell J., Levin S., Anderson C.H., *ApJ*, **609** (2004) 1
- Keisler R. *et al.*, 2011, *Astrophys. J.* 743 28.
- Komatsu E., Seljak, U., 2002, *MNRAS*, **336**, 1256.
- Komatsu E. *et al.* [WMAP Collaboration], 2011, *ApJS*, **192**, 18.
- Larson D. *et al.* [WMAP Collaboration], 2011, *ApJS*, **192**, 16.
- Lewis A., Bridle S., 2002, *Phys. Rev. D*, 66, 103511.
- Lewis A., Challinor A., Lasenby A., 2000, *ApJ*, 538, 473.
- Liu G. -C., Lee S., Ng K. -W., 2006, *Phys. Rev. Lett.*, **97**, 161303.
- Lue A., Wang L. M., Kamionkowski M., 1999, *Phys. Rev. Lett.*, **83**, 1506.
- Lueker M. *et al.*, 2010, *ApJ*, 719, 1045.
- Mather J. C., Fixsen D. J., Shafer R. A., Mosier C., Wilkinson D. T., 1999, *ApJ*, **512**, 511.
- Nolta M. R. *et al.* [WMAP Collaboration], 2009, *ApJS*, **180**, 296.
- O’Dwyer I. J., *et al.*, 2004, *ApJ*, **617**, L99.
- Paci F., Gruppuso A., Finelli F., Cabella C., De Rosa A., Mandolesi N., Natoli P., 2010, *MNRAS*, **407**, 399.
- Page L. *et al.* [WMAP Collaboration], 2007, *ApJS*, **170**, 335.
- Planck Collaboration, ESA publication ESA-SCI (2005)/1, “The Scientific Programme of Planck”, arXiv:astro-ph/0604069.
- Reichardt C. L. *et al.*, 2012, *ApJ*, 749, L9.
- Rocha G., C. R. Contaldi, L. P. L. Colombo, J. R. Bond, K. M. Gorski and C. R. Lawrence, 2010, arXiv:1008.4948 [astro-ph.CO].
- Rudjord O., Groeneboom N. E., Eriksen H. K., Huey G., Gorski K. M., Jewell J. B., 2009, *ApJ*, **692** 1669.
- Slosar A., Seljak U., Makarov A., 2004, *Phys. Rev. D*, **69**, 123003.
- Spergel D. N. *et al.* [WMAP Collaboration], 2007, *ApJS*, **170**, 377
- Tegmark M., de Oliveira-Costa A., 2001, *Phys. Rev. D*, **64**, 063001.
- Verde L. *et al.* [WMAP Collaboration], 2003, *ApJS*, **148**, 195.
- Wandelt B. D., Larson D. L., Lakshminarayanan A., 2004, *Phys. Rev. D* **70**, 083511
- Zaldarriaga M., Seljak U., 1997, *Phys. Rev. D*, **55**, 1830.